

Reply to the comment of Y. Aharonov and L. Vaidman on
“Time asymmetry in quantum mechanics:
a retrodiction paradox”

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Abstract

In the standard physical interpretation of quantum theory, prediction and retrodiction are not symmetric. The opposite assertion by some authors results from their use of non-standard interpretations of the theory.

In their criticism of my Letter [1], Aharonov and Vaidman [2] consider the following situation: “Assume that the x component of the spin of a spin- $\frac{1}{2}$ particle was measured at time t , and was found to be $\sigma_x = 1$. The predictions and retrodictions for the results of σ_x measurements performed after or before the time t are identical. In both cases we are certain that $\sigma_x = 1$.” This claim cannot be derived from standard quantum mechanics.

The correct statement is the following: we can, if we wish, perform another measurement of σ_x at a later time $t_2 > t$, and then we shall indeed find $\sigma_x = 1$ (provided that $H = 0$). On the other hand we *cannot* decide at time t to perform a measurement of σ_x at an *earlier* time $t_1 < t$. At most, we can speculate what would have been the result of such a measurement, if it had been performed. This is counterfactual reasoning, and great care must be exercised. All depends then of what we actually *know* of the state of the system before t . If we know that it was an eigenstate of σ_x , then obviously this was the eigenstate with $\sigma_x = 1$. On the other hand, if we know that it was an eigenstate of σ_y (because the system happens to have been prepared in such a way some time before

t_1) the result of a measurement of σ_x at time t_1 is *random*. It is of course the same as the result of a subsequent measurement of σ_x at time t , but it would be incorrect to assume that the latter must still be $\sigma_x = 1$ if the measurement at time t_1 is actually performed. Indeed, we are considering now two different and mutually exclusive experimental setups: measuring σ_x at times t_1 and t , or only at time t . There is no reason whatsoever to assume that the result obtained at time t is the same for these two different setups.

More generally, if an observer has a partial knowledge of the preparation of a physical system, for example that there are a priori probabilities p_m of having a density matrix ρ_m , and if an ideal measurement of that system produces a definite pure state ψ , we can use Bayesian statistics [3] to deduce posterior probabilities for ρ_m , namely

$$P(\rho_m|\psi) = p_m \langle \psi | \rho_m | \psi \rangle / \sum_n p_n \langle \psi | \rho_n | \psi \rangle.$$

This is the only conclusion that can be legitimately derived from conventional quantum mechanics.

A formal statement of the above property is that an optimal determination of the past of a system can be achieved by an informationally complete set of physical quantities. Such a set is always strongly noncommutative. On the other hand, an optimal determination of the future of a physical system is obtained by a Boolean complete set of quantities [4].

Aharonov and Vaidman [2] also write “Peres encounters a retrodiction paradox because he uses the standard approach ...” I indeed stated in [1] that I was following the conventional “orthodox” quantum formalism, namely the one that is actually used by experimental physicists for analyzing the results recorded by their instruments. This standard interpretation is incompatible with the “time-symmetrized” version of quantum mechanics [5]. Obviously, a different conclusion may be reached by those who prefer to use a different “interpretation” of quantum theory, because that “interpretation” only is a convenient euphemism for what amounts to an essentially different theory.

1. A. Peres, Physics Letters A 194 (1994) 21.
2. Y. Aharonov and L. Vaidman, Physics Letters A [preceding Letter].
3. C. W. Helstrom, Quantum detection and estimation theory (Academic Press, New York, 1976).
4. P. Busch and P. J. Lahti, Found. Phys. 19 (1989) 633.
5. W. D. Sharp and N. Shanks, Philos. Sci. 60 (1993) 488.